

THE IMPORTANCE OF BEING DISCRETE

A Trivial Comedy for Lighthearted Philosophers

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This talk will consist of five parts: Four short allegories and a summary statement. So, with apologies to Oscar Wilde, and your permission...

I The Fox and the Philosopher

Once upon a time there was a very hungry fox. Now Fox was hungry because he could not catch Hare and had not eaten in a long, long time. Fox had tried everything without success, and so, as a last resort, paused from his running hither and thither (mostly thither) to think over his dilemma.

“I must seek the advice of a great man,” concluded Fox to himself after considerable deliberation. “Surely, those wiser than I will tell me how to catch Hare.”

And so it was that Fox came to speak to the most renowned thinker in that world, the Greek philosopher Zeno.

“Sir,” said Fox to Zeno by way of explanation, “I beg your help. I am most surely more cunning than Hare, and yet cannot catch him. On many occasions I have hidden, waiting for Hare to get close so that I could leap out and catch him. But, somehow, when I reach the place where Hare should be, he is no longer. This I do not understand, for I am neither slow nor obvious. Surely, I shall die of hunger unless this great puzzle is solved.”

At this Zeno smiled with delight, for he knew that he understood the problem well. He came to the point straight as an arrow.

“Fox,” said he, “The problem lies in the fact that you seek to catch Hare by overtaking him. You see, by the time you reach the place where last you saw Hare, Hare has moved to a different place, and by the time you reach that place, to yet another place. And this will clearly be true no matter how often you look at Hare or move to catch up with him. Hence, you can never catch Hare by overtaking him. I offer you two solutions to your

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² The present document is a slightly edited version of the original, correcting some typographical and other minor errors.

problem then. You can either find Hare when he will not move, or will come to you while you do not move. Go now, and fill yourself.” And Zeno smiled, pleased with his own great wisdom.

Poor Fox, who truly understood Hare, knew that any attempt to get Hare to wait to be eaten was surely pointless, and certainly would not offer himself up as entrée at Fox’s table. However, out of respect for the great Zeno, he kept these thoughts to himself, thanked Zeno for his wonderful analysis, and bowed out of Zeno’s presence. Weary and greatly discouraged, Fox sat down just outside the entrance to Zeno’s house to think. About that time, a certain student of natural philosophy named Perpetuo happened along. Seeing his old friend Fox in such an obviously bad way, Perpetuo paused to comfort him.

“Fox, old friend. Whatever can be the matter?”

And Fox told Perpetuo of his troubles.

“Well, Fox,” said Perpetuo, “As you know, I never stop seeking after knowledge, and know all the great thinkers. There is a new teacher that you should consult. His thinking surpasses even that of Zeno in its depth and detail.”

And so it was that Fox came to consult the great German thinker Dedekind, widely known for the rationality of his thoughts about irrationality. After Fox told Dedekind the sad story of hunger, of the endless and fruitless chases, and the advice given him by Zeno, Dedekind was glad indeed to help Fox.

“Yes,” said Dedekind, “Zeno is essentially correct, but for all the wrong reasons. You see, Fox, you must consider the path to be covered³ before you chase hare. If you were to cut the path into two parts, you must clearly cross the boundary between the two parts to reach Hare. And if you cut either of these two parts in to, then you must cross the boundary induced by that cut as well. Clearly, this process of cutting the parts of the path into two pieces can be continued forever. Therefore, there will always be an infinite number places you must be first before you catch Hare and, of course, this is not possible.”

At this pronouncement, Fox was greatly distressed and started to speak, but Dedekind stopped him...

“Wait, my friend, all is not lost for in this very fact we find a solution to your dilemma. Certainly, this fact of nature is as true for Hare as for you. Hare can never reach a place of safety that he has not already reached. Therefore, you need only explain this to Hare next time you see him and he will surely give up his running away.”

Now Fox was somewhat confused by the clarity (or irrationality) of Dedekind’s reasoning, but he was delighted with the prospect it offered. So he thanked Dedekind profusely for saving his life and ran off to find Hare.

³ That is, the path must have a cover.

Knowing Hare as well as he did, this did not take long. Fox came close to finding Hare, for Hare was on one side of a stream that cut across a trail through the forest, and Fox was on the other. Hare listened to Fox patiently while Fox called out to him from across the stream, telling him with great eloquence the mysteries surround running after (or approaching from below) and running away (or retreating from above) without limit.

Finally, Hare could stand it no longer and began to laugh so hard his hind legs thumped the ground furiously. And Hare bounded away, laughing while the bewildered and disappointed Fox called after him. (To this day, hares laugh whenever they think of Fox. You can tell by the way their hind legs thump the ground whenever they realize they are being chased!)

Fox tried many times that day to convince Hare he was right. In fact, he repeated the argument often, but was never able to finish it enough to count. The result was always the same: Hare laughed, thumped, and ran away. Tired and weak from hunger, Fox lay down beside the road to rest and consider his plight. About this time, along came Perpetuo.

“So, Fox!” What success?”

Fox told Perpetuo of all that had transpired. Perpetuo could see that Fox was almost without hope and was desperate.

“Well, Fox, I think I have just the thing. Let’s go talk to the Lighthearted Philosopher. I am sure he can help, for though not many have heard of him, he is highly original and sometimes offers the most amazing solutions to problems such as yours.”

And so they came to visit the Lighthearted Philosopher (LP to his friends). Perpetuo spoke for Fox who was very weak indeed, telling LP the sad story of hunger, of the endless and fruitless chase, the pointed advice given him by Zeno, and of the Dedekind cut. LP understood how Zeno and Dedekind had put the problem in the neighborhood of a real Hare-ball for Fox. Still, he instantly comprehended the problem and, of course, how to solve it. (Amazing how philosophers can do that!)

“Fox, a great hunter such as you must have seen that Dedekind’s analysis of the problem was irrational, and hence flawed. Surely you know that to focus on the path instead of the prey is a mistake indeed. You must learn to think outside the box. We live in a world of four dimensions, not three. Time is naught but another path. Wherever Hare has been, and will be, is but a different place in spacetime, like unto being here and there. Hare has no time-varying position, only existence. Seen in four-space, Hare is like the path, being at all locations. And Dedekind is correct, any path can be split and split again, continuously. Why, even a Hare path! Thus, there is as much Hare there as here, and here as much Hare as there. So, wherever you are on the Hare path, you have caught the Hare that was (or will be) there!”

Hope sprang eternal (or at least continuously) in Fox. Suddenly, he saw it all. In his enlightened and lighthearted state, he forgot his hunger and felt so much better, that he became even more convinced he had truly found a solution to his problem. Like his kind ever since, he grew quite lean. – who among you has ever seen a fat fox? – for thereafter, as before, Fox only ate when he forgot himself.

The solution of the Lighthearted Philosopher achieved great fame and spread far and wide. The Lighthearted Philosopher's analysis seemed so natural, he came to be known as a natural philosopher, and natural philosophers never worry about combining the rational and the irrational. And there were continuous discussions concerning the wonder of it all. But we would have to be discrete to say who had these discussion or more of their subject, and would involve splitting Hares indeed.

II Royal Arithmetic

Long ago, in a land far, far away, there was a great King. Now the Magistrate over the King's orchards, one Benjamin of poor judgment, liked to wager so much it frequently got him into considerable, almost denumerable, difficulties. This was just such an occasion. It seems that the Magistrate Benjamin had lost a wager with the Emperor from a neighboring land. The Kingdom's riches were the stakes, namely five thousand gallons of fruit juice, to be paid in fruit. This accounted for exactly one third of the Kingdom's fruit harvest for a normal year. Unfortunately, there had been a bad frost in the spring following the wager and two thirds of the fruit were frost damaged on the leeward side. If the wager was paid with the good fruit needed to make five thousand gallons of fruit juice, the country would most assuredly be bankrupt.

But the great and wise King did not despair. Being of a legal bent of mind (or perhaps a legally bent mind⁴), he called on the Emperor. Explaining that he needed a bit of time to prepare payment for his Magistrate's debt, the King asked the Emperor to accept a short term I.O.U. for the fruit due. In exchange for this consideration he offered to cut all the fruit in half, so that the Emperor would have less work to do in making the fruit juice. To this the Emperor readily agreed, on the condition that the I.O.U. state the exact count of fruit to be delivered. To this the King objected, pointing out that payment would be made in half fruit, the I.O.U. should be worded accordingly. The Emperor and the King came to agreement and the I.O.U. was written, with the Emperor acknowledging that payment in half fruit eliminated all obligations owed him by the King's Magistrate.

As you can imagine, the clever King proceeded to have Magistrate Benjamin cut in half the fruit that had been blessed by the God of Fruit, and gave the bad halves to the Emperor. Thus was the debt satisfied, for the Emperor had been so concerned with quantity, that the I.O.U. had failed to consider quality. Of course, bad fruit, even half fruit, led to bad fruit juice, but that was not the half of it.

In time, the Emperor's kingdom so weakened from drinking the bad fruit juice, that the wise King conquered them with ease. The Emperor and his successors learned a lesson on that day from the wise King. And it came to pass that they taught that lesson to the Romans when they invaded the land generations later. So it was that the wise King was surely responsible for two legendary interpretations of that lesson, for he taught the Romans to divide and conquer, and his own people to divide and multiple.

⁴ Isn't "legal mind" an oxymoron? Or perhaps having a legally bent mind (or bent legal mind) is repetitious...

III Counting Down

There was once a great Republic in which, though everyone could add, subtract, divide, and multiple, no one could count (well, at least, hardly anyone counted). In this great land there was a new President.⁵ The President's predecessors, who also could not count and were in fact not accountable, had been very extravagant. They had spent lots of money on nuclear and other armaments because they were afraid of the leaders of another great county. Now this had resulted in a huge national debt which, due to deficit spending, was growing larger each day. But the President thought that he had a solution to the problem. So he called his advisors together.

"Gentlemen, I wish to know if we can reduce the national budget by not buying more nuclear arms."

The advisors began to protest, pointing out that they would surely be invaded by the enemy, who had often stated that they had sufficient nuclear arms for their purposes. And, of course, everyone knew the enemy's evil purpose.

"But, surely," countered the President, "we can afford to decrease the spending on nuclear arms a bit, because we have more than enough to destroy everyone on the planet many times over and we certainly have no desire to destroy our own people. And if we decrease the spending a bit, will we still not have enough power to destroy everyone on the planet many times over?"

To this the advisors had to agree, so the President continued: "I charge you then, go and determine by how much I can decrease the spending, for surely if I do not decrease the spending, the country will be bankrupt and if I decrease too much, the country will be invaded."

So the advisors – especially those wise in economic matters – went to work. And they worked continuously, adding and subtracting, multiplying and dividing one bit at a time as discreetly as possible. Eventually, continuing the President's line of thinking, they took the process to the very limits of the President's patience. In due course they returned to the President with an answer.

And in time it came to pass that the President followed their advice and spent nothing on arms, thus saving his country from bankruptcy. Everyone realized that if he hadn't, eventually both their country and the enemy country would have gone bankrupt and ceased to exist. And so the country was not invaded because the enemy, who could count, knew about zero and infinity. And zero arms were certainly sufficient for their purpose, for then overkill could be accomplished for, why next to nothing. But then, this tale had to have a good ending or else no one would have counted.

⁵ Well, actually he was kind of old and had been elected before, so he wasn't very new at all.

IV The Importance of Being Discrete

This is a story about a fateful romance between a man and a woman of differing ideologies, Kathryn and Earnest. She lived in a safe, closed neighborhood with nice clear boundaries. She knew each and every one of her neighbors and could easily enumerate them. Why, the neighborhood was so sparse, in fact, that one could hardly call it a neighborhood. He lived in an open neighborhood with more neighbors than he could count (in fact, they were denumerable). Earnest was steadfast, friendly, open, honest, all in all a nice young man. Except that Earnest had one flaw – he talked continuously about everything. And I do mean everything. But eventually, and paradoxically, it was his continual flattery of Kathryn that won her.

Now after he bedded Kathryn, it did not take long before the town began to whisper about Kathryn's indiscretion. And this got back to her. Of course, she was outraged. Why, she was the very soul of discretion. How could he have been so indiscrete? After she had been so careful, so cautious, so... well, discrete.

As you might have guessed, this couple was headed for disaster. After all, there is no way to establish a one-to-one relationship between the continuous and the discrete, so they were truly matchless... uh... even after the continuous had been embedded with the discrete.

* * *

Mathematics is filled with hidden infinities, notions of continuity, limits, open neighborhoods, and even randomness. Our first story, *The Fox and the Philosopher*, illustrates the danger of assuming continuity while performing digital operations. No matter how sophisticated the sophistry, paradoxes are bound to result due to the unacknowledged infinities. Like the Fox, continuum philosophers' propositions are often not satisfiable, and their remedies seldom satisfying.

The second tale, *Royal Arithmetic*, demonstrated a subtlety of continuum mathematics. The notion of doing arithmetic with decimal fractions is deceptive and seductive. It is essentially incorrect in a discrete world. An orange is not the same as two half oranges any more than two halves of the child that would have resulted from King Solomon's wisdom are the same as one child. Partitioning discrete objects (e.g., sets, spaces, and so on) yields objects of a class different from that of the original objects. But continuum mathematics makes no distinctions, and no provision for preserving distinctions of class. If this is true in the discretum, why should it not be true in the construction and analysis of the continuum (as proposed by, for example, Dedekind)? The assumption is made that all qualities are equally distributed, are therefore equally divisible, and furthermore have no fundamentally smallest unit. This notion carries over into interpretations of statistics and randomness. Indeed, it is not the same to refer to "one out of every five" as to "two out of every ten." Knowing two out of every ten tells us nothing about the partitioned

set's distribution, so we are not assured that any property will carry over to the one out of very five. And, of course, the definition of randomness involves infinity – either in the continuum or a denumerable infinity – which is itself not definable constructively.

The third tale, Counting Down, illustrates the ill-conceived notion of doing discrete subtraction from a continuous quantity. The operation is clearly incompatible with the continuum concepts involved in the President's analysis. It is this same mixing of the continuous and the discrete that has led some to postulate "fuzzy logic" – with the obvious result that all the mathematics and concepts therein become "fuzzified."

Our last effort, from which the title of the talk was taken, attempts to point out in a gentle way the incompatibility between the continuous and the discrete. Though counting, measurement, and arithmetic often appear in close association in mathematics, physics, philosophy, etc., etc., these have no clearly defined mathematical association without additional assumptions which are usually unacceptable in the given context. For example, there is no definition of mapping or function without the notion of an open neighborhood and discrete spaces can have no open neighborhood. Without a definition of function, there is no distance function and no metric. We do not mean to imply that a mathematics could not be constructed which would eliminate these problems, only that the accepted foundations of mathematics does not serve the need.

Understanding this situation is particularly important in logic, empirical modeling, psychology, and quantum physics. It is all too easy to assume that all is well in the foundations, only to labor intensively trying to resolve the paradoxes that must result in some abstruse form.